Solutions to Problem 1.

a The diagonal entries of $\mathbf{P}$ are the probabilities that a consumer purchases a brand, given that they previously purchased the same brand. The diagonal entries being higher than the off-diagonal entries indicates that a consumer is more likely to stick with a brand if they previously purchased that brand.

b The initial state vector is

$$\mathbf{p}^\top = \begin{bmatrix} 1/2 & 1/2 & 0 \end{bmatrix}$$

We want $p_3^{(50)}$.

$$\mathbf{p}^{(50)\top} = \mathbf{p}^\top \mathbf{P}^{50} \approx \begin{bmatrix} 0.455 & 0.455 & 0.091 \end{bmatrix}$$

So, $p_3^{(50)} \approx 0.091$.

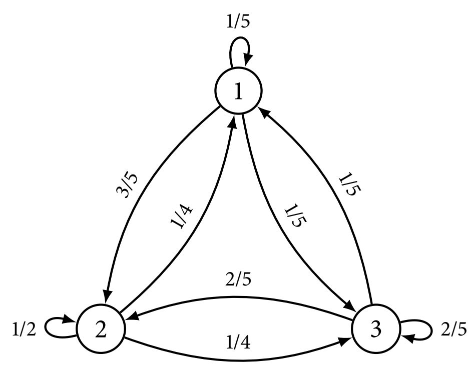
c Let $\mathcal{A} = \{1, 2\}$, $\mathcal{B} = \{3\}$. We want $f_{23}^{(10)}$.

$$\mathbf{F}_{\mathcal{AB}}^{(10)} = \mathbf{P}_{\mathcal{AA}}^9 \mathbf{P}_{\mathcal{AB}} = \begin{bmatrix} 0.70 & 0.28 \\ 0.28 & 0.70 \end{bmatrix}^9 \begin{bmatrix} 0.02 \\ 0.02 \end{bmatrix} \approx \begin{bmatrix} 0.017 \\ 0.017 \end{bmatrix}$$

Therefore, $f_{23}^{(10)} \approx 0.017$.

Solutions to Problem 2.

a.



b. We want $\Pr\{S_3 = 1 \mid S_0 = 1\} = p_{11}^{(3)}$.

$$\mathbf{P}^{(3)} = \mathbf{P}^3 = \begin{bmatrix} 0.225 & 0.496 & 0.279 \\ 0.225 & 0.495 & 0.280 \\ 0.224 & 0.492 & 0.284 \end{bmatrix}$$

So, $p_{11}^{(3)} = 0.225$.

c. Since the AGV is equally likely to be at any of the three locations, the initial state vector is

$$\mathbf{p}^\top = \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix}$$

We want $\Pr\{S_3 = 3\} = p_3^{(3)}$.

$$\mathbf{p}^{(3)\top} = \mathbf{p}^\top \mathbf{P}^3 \approx \begin{bmatrix} 0.2247 & 0.4943 & 0.2810 \end{bmatrix}$$

So, $p_3^{(3)} \approx 0.2810$.

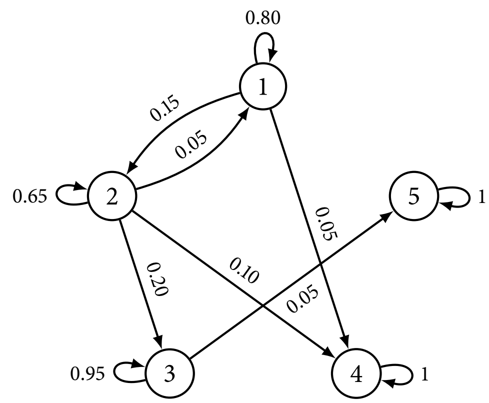
d. Let $\mathcal{A} = \{1, 2\}$ and $\mathcal{B} = \{3\}$. We want $f_{23}^{(5)}$.

$$\mathbf{F}_{\mathcal{AB}}^{(5)} = \mathbf{P}_{\mathcal{AA}}^{4}\mathbf{P}_{\mathcal{AB}} = \begin{bmatrix} 1/5 & 3/5 \\ 1/4 & 1/2 \end{bmatrix}^4 \begin{bmatrix} 1/5 \\ 1/4 \end{bmatrix} \approx \begin{bmatrix} 0.0839 \\ 0.0790 \end{bmatrix}$$

So, $f_{23}^{(5)} \approx 0.0790$.

Solutions to Problem 3.

a.



b. The probability that a lawyer leaves as non-partner, given that the lawyer left as a non-partner in the previous year is 1. This value is p_{44}. Likewise, the probability that a lawyer leaves as a partner, given that the lawyer left as a partner in the previous year is 1. This value is p_{55}.

c. We want $\Pr\{S_5 = 3 \mid S_0 = 1\} = p_{13}^{(5)}$.

$$\mathbf{P}^{(5)} = \mathbf{P}^5 \approx \begin{bmatrix} 0.3597 & 0.2176 & 0.1572 & 0.2546 & 0.0109 \\ 0.0725 & 0.1422 & 0.4473 & 0.2711 & 0.0669 \\ 0 & 0 & 0.7738 & 0 & 0.2262 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

So, $p_{13}^{(5)} \approx 0.1572$.

d. We want $\Pr\{S_5 = 3\} = p_3^{(5)}$.

$$\mathbf{p}^{(5)T} = \mathbf{p}^T\mathbf{P}^5 \approx \begin{bmatrix} 0.2843 & 0.1916 & 0.2460 & 0.2452 & 0.0329 \end{bmatrix}$$

So, $p_3^{(5)} \approx 0.2460$.

e. Let $\mathcal{A} = \{1, 2\}$ and $\mathcal{B} = \{4\}$. We want $f_{14}^{(6)}$.

$$\mathbf{F}_{\mathcal{AB}}^{(6)} = \mathbf{P}_{\mathcal{AA}}^{5}\mathbf{P}_{\mathcal{AB}} = \begin{bmatrix} 0.80 & 0.15 \\ 0.05 & 0.65 \end{bmatrix}^5 \begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix} \approx \begin{bmatrix} 0.0397 \\ 0.0178 \end{bmatrix}$$

So, $f_{14}^{(5)} \approx 0.0397$.